Hence, a distortionless stripline, which is reflectionless for right-moving waves, should have an exponentially tapered characteristic impedance that increases if $G_0 Z_0 > R_0 Y_0$ and decreases if $G_0 Z_0 < R_0 Y_0$. The power gain follows from a derivation that is similar to that in (23) and (24), but both R and G are different from zero here. The result is

$$\frac{P_t}{P_i} = \exp\left[-(G_0 Z_0 + R_0 Y_0) \cdot l\right]. \tag{28}$$

V. Discussion

A new condition for distortionless nonuniform transmission lines has been developed that is a generalization of the Heaviside distortionless condition. The derivation uses the wave-splitting technique, and it is carried out in the time-domain. It is shown how the distortion can be eliminated by matching the series resistance and shunt conductance to the slope of the characteristic impedance. One should notice that the model assumes that the transmission line parameters are nondispersive. This means that if one cannot neglect dispersion, the distortionless condition can only be made valid for a limited band of frequencies. The conditions imposed on the transmission line parameters are that R, G and the slope of Z are piece-wise continuous.

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New Model of Coupled Transmission Lines

Adam Abramowicz

***Abstract*—The paper shows that an existing description of coupled transmission lines is inconsistent and proposes a new model based truly on the mutual coupling concept. In the existing formulation a series electric coupling and parallel magnetic coupling are combined. In the new formulation the parallel electric and magnetic couplings as well as series electric and magnetic couplings are used. Obtained model of coupled lines has physical background related to the odd and even type of propagation and agrees with the practical results.**

I. Introduction

When two unshielded uniform TEM transmission lines of the same impedance Z are located in close proximity, they become electromagnetically coupled via their associated electric and magnetic fields. Two coupled lines can be excited in the two ways: "even mode" excitation or "odd mode" excitation, i.e., in-phase or opposite-phase, equal-amplitude excitations. The characteristic impedances Z_{0e} and Z_{0o} associated with these modes are defined as the input impedance of an infinite length of one line, in the presence of (and thus electromagnetically coupled to) the second line, also of infinite length, when both are excited in the appropriate manner. A knowledge of Z_{0e} and Z_{0o} as functions of line parameters is essential to the design of filters, directional couplers, and related devices, because the coupling coefficient between lines can be calculated from them. As it has been shown in [1] the coupling coefficient k between two coupled lines when they are properly terminated can be calculated from the following formula:

$$k = \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}}. \tag{1}$$

Lines are properly terminated when the matching impedance Z_0 is taken as

$$Z_0 = \sqrt{Z_{0e} Z_{0o}}. \tag{2}$$

The impedance Z_0 is always less than the impedance of single line Z (without coupling), thus four mentioned impedances satisfy the following inequality:

$$Z_{0o} < Z_0 < Z < Z_{0e}. \tag{3}$$

All four impedances can be simply expressed in terms of the capacitance per unit length of the particular transmission line in question: if this parameter is denoted by C (F/m), then

$$Z\sqrt{\varepsilon_r} = \frac{1}{vC} \tag{4}$$

where: v is the velocity of light in free space and ε_r is the dielectric constant of the medium filling the line.

It should be also noted that for the uniform coupled lines the velocity of light is the same for odd or even excitations and equal to the velocity of light in the single (uncoupled) line.

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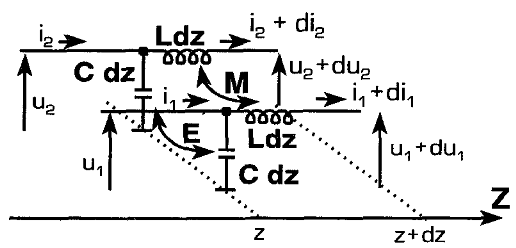

Fig. 1. Lumped element representation of the elemental length of coupled lines.

II. Existing Description of Coupled Lines [2]

In the traditional description of two symmetrical coupled transmission lines [1], [2] the following set of differential equations is assumed (see Fig. 1):

$$\begin{aligned}\frac{\partial u_1}{\partial z} &= -L_1 \frac{\partial i_1}{\partial t} - M \frac{\partial i_2}{\partial t}\\ \frac{\partial u_2}{\partial z} &= -M \frac{\partial i_1}{\partial t} - L_1 \frac{\partial i_2}{\partial t}\\ \frac{\partial i_1}{\partial z} &= -C_1 \frac{\partial u_1}{\partial t} + E \frac{\partial u_2}{\partial t}\\ \frac{\partial i_2}{\partial z} &= +E \frac{\partial u_1}{\partial t} - C_1 \frac{\partial u_2}{\partial t}\end{aligned} \tag{5}$$

where

- L_1 is an inductance per unit length of the single line with the presence of second line taking u and i in the second line to be zero,
- C_1 is the capacitance per unit length of the single line with the presence of second line taking u and i in the second line to be zero,
- M is the mutual inductance of coupled lines per unit length,
- E is the mutual capacitance between lines per unit length

As in any system involving coupled linear circuits, we have a set of simultaneous linear differential equations which admits a particularly simple set of solutions: the normal modes of the system. In this case we have four normal modes (two in each direction) characterized by fixed ratios of u_2/u_1 and i_2/i_1 which do not change as the wave propagates. In our case the modes are characterized by

$$\begin{aligned}\frac{u_2}{u_1} &= \pm 1\\ u_1 i_1 &= u_2 i_2.\end{aligned} \tag{6}$$

The choice of the plus sign corresponds to the even mode excitation, the minus sign to the odd mode excitation. In either normal mode equal power is associated with each line. Letting

$$k_C = \frac{E}{C_1}, \qquad k_L = \frac{M}{L_1}. \tag{7}$$

Equation (5) reduces to the familiar transmission line equations

$$\begin{aligned}\frac{\partial u_j}{\partial z} + L_1(1 \pm k_L) \frac{\partial i_j}{\partial t} &= 0\\ \frac{\partial i_j}{\partial z} + C_1(1 \mp k_C) \frac{\partial u_j}{\partial t} &= 0\end{aligned} \tag{8}$$

where $j = 1, 2$.

From these the velocity of propagation

$$v = \frac{1}{\sqrt{L_1 C_1 (1 \pm k_L)(1 \mp k_C)}}. \tag{9}$$

Because v must be the same for all modes $k_L = k_C = k$

$$v = \frac{1}{\sqrt{L_1 C_1 (1 - k^2)}}. \tag{10}$$

Because the velocity of propagation must be independent of k, thus L_1 and C_1 must vary as $(1-k^2)^{-1}$. Oliver [2] has made a groundless assumption that

$$L_1 \cong L, \qquad C_1 \cong \frac{C}{1-k^2} \tag{11}$$

where L and C are the inductance and capacitance of the single uncoupled line

As a result the impedances of the even and odd mode can be expressed

$$\begin{aligned}Z_{0e} &= Z(1+k)\\ Z_{0o} &= Z(1-k)\end{aligned} \tag{12}$$

and the impedance of a single (uncoupled) line can be expressed by Z_{0e}, Z_{0o} and k as

$$Z = \sqrt{Z_{0e} Z_{0o}} \frac{1}{\sqrt{1-k^2}}. \tag{13}$$

The coupling coefficient k—(12)—is exactly the same as (1). Obtained (12), (13) are in a fair agreement with the practice (they suffer accuracy for tight couplings) but they have been obtained under the unjustified assumption (11).

Looking back at the sets (5) and (8), it is easy to recognize that they describe the mutual parallel coupling of two inductances and the mutual capacitive series coupling of two capacitances which are well known [3]–[6] (by the way, they appear in the paper [2] but two paragraphs later) at least as the impedance and admittance inverters [6], and are often applied to the coupled lines description [4], [6].

In the mutual parallel magnetic coupling $L_1 = L$, while in the capacitive series coupling $C_1 = C$. The definitions of the coupling coefficients are expressed as

$$k_C = \frac{E}{C}, \qquad k_L = \frac{M}{L}. \tag{14}$$

Assuming that $k_L = k_C = k$ the velocities of propagation can be obtained from (8)

$$v = \frac{1}{\sqrt{LC(1-k^2)}} \tag{15}$$

that are clearly different from the velocity of propagation of single uncoupled line which is

$$v = \frac{1}{\sqrt{LC}}. \tag{16}$$

Consequently the even and odd mode impedances can be expressed as

$$\begin{aligned}Z_{0e} &= Z\sqrt{\frac{1+k}{1-k}}\\ Z_{0o} &= Z\sqrt{\frac{1-k}{1+k}}\end{aligned} \tag{17}$$

and the impedance of a single (uncoupled) line is

$$Z = \sqrt{Z_{0e} Z_{0o}}. \tag{18}$$

The coupling coefficient k resulting from (17) is again the same as (1).

Equations (15), (17), and (18) describing Oliver's model of coupled lines [2] cannot be accepted because the velocities of propagation for the even and odd modes are different than that for the single uncoupled line.

To briefly sum up the Oliver's theory it must be said that his equations are inconsistent and his theory is mistaken. Although (1), (12), and (13) are in fair agreement with the factual data, they have been obtained through the misinterpretation of the initial system (5) and the groundless assumption (11).

III. New Model of Coupled Lines

In the proposed new model both electric and magnetic mutual couplings are used twice: in the parallel and series coupling. The model is based on a physical phenomenon of coupled lines behavior. When two lines are situated in an infinite distance they are uncoupled. While the distance is decreased the lines become coupled and two types of an excitation are possible. The even and odd mode impedances relate to these excitations. The values of the Z_{0e} and Z_{0o} depend on the coupling coefficient or strictly speaking the distance between lines. In the coupled state the self inductance and capacitance of lines are changed according to the strength of coupling, thus these changes depend on the coupling coefficient. *It should be added that they depend on the coupling coefficient only* . A set of differential coupled transmission lines equations describing two (electric and magnetic) parallel couplings and two (electric and magnetic) series couplings can be written

$$\begin{aligned} \frac{\partial u_j}{\partial z} + L\,\frac{1 \pm k_{Mp}}{1 \mp k_{Ms}}\,\frac{\partial i_j}{\partial t} &= 0 \\ \frac{\partial i_j}{\partial z} + C\,\frac{1 \mp E_s}{1 \pm k_{Ep}}\,\frac{\partial u_j}{\partial t} &= 0 \end{aligned} \tag{19}$$

where

L	is an inductance per unit length of the line with the absence of second line,
C	is the capacitance per unit length of the line with the absence of second line,
k_{Ms}	is the series magnetic coupling coefficient,
k_{Es}	is the series electric coupling coefficient,
k_{Mp}	is the parallel magnetic coupling coefficient,
k_{Ep}	is the parallel electric coupling coefficient,
$j = 1, 2$	

The coupling coefficients k_{Ms} and k_{Es} for the series couplings are defined by the following relations [7], [8]

$$k_{Es} = \frac{E_s}{C}, \qquad k_{Ms} = \frac{L}{M_s} \tag{20}$$

where

M_s	is the series mutual inductance of coupled lines per unit length,
E_s	is the series mutual capacitance between lines per unit length

while the coupling coefficients k_{Mp} and k_{Ep} for the parallel couplings are defined by the complementary relations [7], [8]:

$$k_{Ep} = \frac{C}{E_p}, \qquad k_{Mp} = \frac{M_p}{L} \tag{21}$$

where

M_p	is the parallel mutual inductance of coupled lines per unit length,
E_p	is the parallel mutual capacitance between lines per unit length

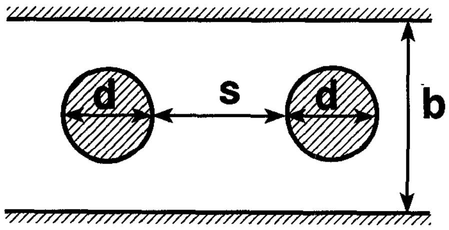

Fig. 2. Configuration of coupled slab-lines.

subsequently, from (19) the velocities of propagation

$$\begin{aligned} v_e &= \frac{1}{\sqrt{LC\,\dfrac{(1+k_{Mp})(1-k_{Es})}{(1+k_{Ep})(1-k_{Ms})}}} \\ v_o &= \frac{1}{\sqrt{LC\,\dfrac{(1-k_{Mp})(1+k_{Es})}{(1-k_{Ep})(1+k_{Ms})}}} \end{aligned} \tag{22}$$

which are equal to the velocity of propagation of the single uncoupled line when

$$k_{Mp} = k_{Ep} = k_p$$

and

$$k_{Ms} = k_{Es} = k_s$$

The impedances of the even and odd mode are

$$\begin{aligned} Z_{0e} &= Z\,\frac{1+k_p}{1-k_s} \\ Z_{0o} &= Z\,\frac{1-k_p}{1+k_s} \end{aligned} \tag{23}$$

and the impedance of a single (uncoupled) line Z can be expressed by Z_{0e}, Z_{0o} and k_p, k_s as

$$Z = \sqrt{Z_{0e}Z_{0o}}\,\frac{\sqrt{1-k_s^2}}{\sqrt{1-k_p^2}}. \tag{24}$$

The coupling coefficients k_s and k_p found from (23) are as follows:

$$\begin{aligned} k_s &= \frac{Z_{0e} + Z_{0o} - 2Z}{Z_{0e} - Z_{0o}} \\ k_p &= \frac{Z(Z_{0o} + Z_{0e}) - 2Z_{0e}Z_{0o}}{Z(Z_{0e} - Z_{0o})}. \end{aligned} \tag{25}$$

The total coupling coefficient k in the most interesting case of two quarter wave length coupled lines terminated by the impedances Z_0 found according to the equation (2) can be expressed by the coupling coefficients k_s and k_p as:

$$k = \frac{k_p + k_s}{1 + k_p k_s}. \tag{26}$$

The coupling coefficient k calculated according to (26) is the same as calculated from (1).

IV. An Example of Coupled Lines

As an example two coupled slab lines are analyzed. The impedances of a single line (Z) and coupled lines (Z_{0e} and Z_{0o}) are taken from [9]. In fact in [9] the capacitances of lines are given but from (4) the impedances can be immediately found. These impedances will be used to check the consistency of the old and new model of coupled lines. The configuration of two-conductor slab-lines is presented in Fig. 2. The parameters of the structure, impedances of coupled lines Z_{0e} and Z_{0o}, coupling coefficients k, k_p, and k_s, and the impedances of a single uncoupled line Z calculated from (14) and (25) are presented in Table I.

TABLE I
IMPEDANCES AND COUPLING COEFFICIENTS OF COUPLED SLAB-LINES (d/b = 0.4; THE IMPEDANCE OF A SINGLE ROD BETWEEN PARALLEL PLANES IS 69.30 Ω)

s/b	Z_{0e} / Ω	Z_{0o} / Ω	k	k_s	k_p	Z(14) / Ω	Z(25) / Ω
0.08	90.44	33.52	.4592	-.2573	.6408	61.98	69.30
0.12	88.35	39.60	.3810	-.2186	.5536	63.97	69.30
0.16	86.43	44.23	.3329	-.1882	.4818	65.33	69.30
0.20	84.68	47.93	.2771	-.1631	.4211	66.31	69.30
0.24	83.08	50.96	.2396	-.1421	.3692	67.02	69.30
0.40	78.04	58.95	.1393	-.08447	.2212	68.50	69.30
0.60	74.13	64.01	.07326	-.04625	.1191	69.07	69.30
0.76	72.27	66.15	.04415	-.03028	.07434	69.21	69.30

The results presented in Table I clearly show that although the coupling coefficient between lines resulting from the old model is the same as that obtained from the new model, the new model exactly describes relations between the impedances Z, Z_{0e} and Z_{0o}, and the coupling coefficients k, k_p, and k_s, while the old model is inconsistent (see column 7 in Table I).

The coupling coefficients k_p and k_s are the "internal" coupling coefficients that depend only on the distance between coupled lines and their dimensions. The "external"—overall coupling coefficient k depends on the impedances terminating coupled lines.

The parallel coupling coefficient k_p can be related to the even mode of propagation (the electromagnetic fields are in phase) and the series coupling coefficient k_s (the sign "−" in Table I) can be related to the odd mode of propagation (the electromagnetic fields are out of phase). The electromagnetic field of coupled lines is a superposition of two field distributions: the even and odd mode.

For small couplings ($k < 0.1$) (26) can be simplified to

$$k = k_p + k_s. \tag{27}$$

V. CONCLUSION

The Oliver's equations (12) and (13) have been used for years because they fairly well agree with the practice, but they have been obtained from the false model and groundless assumption. The presented new model relates the coupling coefficient between transmission lines to the odd and even types of excitation, thus to the electromagnetic field distribution. Four coupling elements and consequently four coupling coefficients are used to describe coupling. The series capacitor E_s and series inductance M_s are responsible for the series coupling related to the odd mode of propagation. The shunt (parallel) capacitor E_p and shunt (parallel) inductance M_p are responsible for the parallel coupling related to the even mode of propagation. The electric coupling coefficient and magnetic coupling coefficient are equal in both series and parallel types of coupling, accordingly the series and parallel coupling are described with only one coupling coefficient k_s or k_p respectively. The total coupling coefficient results from two coupling coefficients of series k_s and parallel k_p couplings.

Obtained equations describing relations between impedances Z_{0e}, Z_{0o}, Z and the coupling coefficients k_s and k_p are in excellent agreement with the theory described in the introduction and with the experimental results even for tight couplings. The main disadvantage of the Oliver's model—unequal velocities of propagation has been eliminated.

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